Faculty of Engineering
Electrical and Electronics Engineering Department
EE 303 Numerical Techniques and Programming

Answer all questions to the best of your knowledge. Carry all calculations up to 3 decimal places.

Time allowed: 2 hrs.

Q1.

a) Find the Taylor series expansion for the function $f(x) = \ln\left(\frac{1+x}{1-x}\right)$ **(2.5 Marks)**

b) Write a recursive expression of the form $T_{i+1} = (\ldots)T_i$ for the Taylor series expansion of the function given in part a of the question. **(2.5 Marks)**

c) Apply Newton's method to equation $x^2 = N$ to derive the algorith for getting the square root of N. **(2.5 Marks)**

d) Write a c program to find the root of a non-linear equation using secant method. **(2.5 Marks)**

Q2.

Use Newton's method to find the roots of the following non-linear system

$f(x,y) = 4 - x^2 - y^2$

$g(x,y) = 1 - e^x - y$

Start at $x_0 = 1, y_0 = -1.7$, perform only 3 iterations. **(10Marks)**

Q3.

Given

$$A=\begin{bmatrix} -4 & -12 & 3 \\ 10 & -4 & 5 \\ -5 & 2 & 6 \end{bmatrix}, b=\begin{bmatrix} -46 \\ 4 \\ 15 \end{bmatrix}$$

a) Solve the system using Gaussian elimination method with partial pivoting. **(5 Marks)**

b) Show that same solution can be obtained using Cramer's rule. **(5 Marks)**

Q4.

For the matrix given in q3.,

a) Show that *det(A)=det(L)*det(U)* **(5 Marks)**

b) Show that the matrix is not *ill-conditioned.* (Use Euclidean Norm) **(5 Marks)**

Good luck to all of you

Faculty of Engineering
Electrical and Electronics Engineering Department
EE 303 Numerical Techniques and Programming
Midterm I, May 2nd, 2009

a) Answer all questions to the best of your knowledge.
b) Show all steps and carry all calculations up to 3 digits unless otherwise mentioned.
c) No question will be answered during the exam.
d) Time allowed: *2 hours*

Q1-

(a) Derive the Taylor series at 0 for the function $f(x) = \ln(x + 1), x \neq -1$,
(b) Write the series summation notation.
(c) Find a recursive formula of the form $T_n=(\ldots)\, T_{n-1}$ for the function in part (a) of this question.
(d) Using f(1.5). How many terms are needed for the absolute error to drop down to 10^{-2}, given the true value= 0.585
(e) Write a c/c++ program to compute the series in part c of this question

0.04

$\left|\frac{TV - \ldots}{TV}\right|$

0.00

Q2-

(a) Derive the formula for the secant method
(b) Using Newton-Raphson method, Show that the root of the function $f(x) = \sqrt[m]{N}$ can be written as

$$x_{n+1} = \frac{(m-1)x_n^m + N}{m\, x_n^{m-1}}$$

(c) Using the half-interval method, find the root of $f(x) = 2 * \sin(x) - e^{x/4}$ (4 iterations)

Q3.

(a) Define the following: Diagonally dominant matrix, ill-conditioned system, partial pivoting, singular matrix.
(b) Give two Matlab commands for finding the condition number of a matrix.
(c) Use *LU* factorization method to solve the following system

$$2x + y - 3z = -11$$
$$3x - 4y + 5z = 38$$
$$-2x + 3y + 7z = 15$$

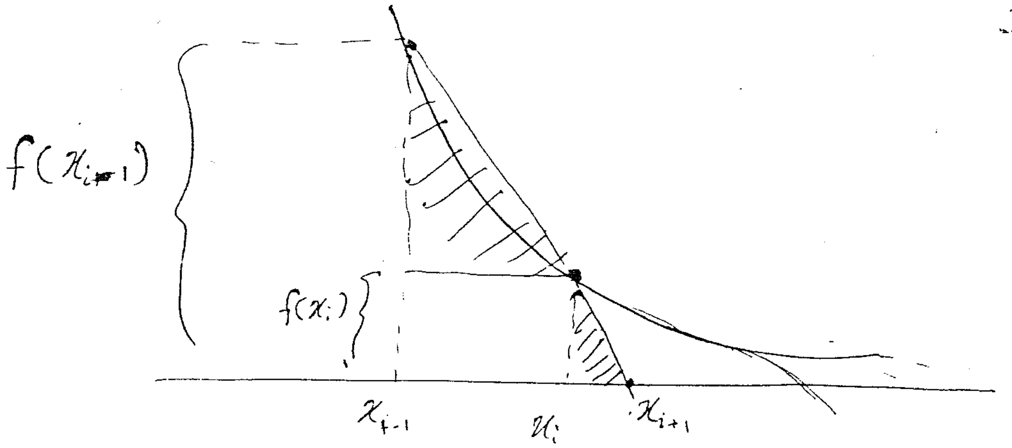
Good Luck to all of you

2
-3
4

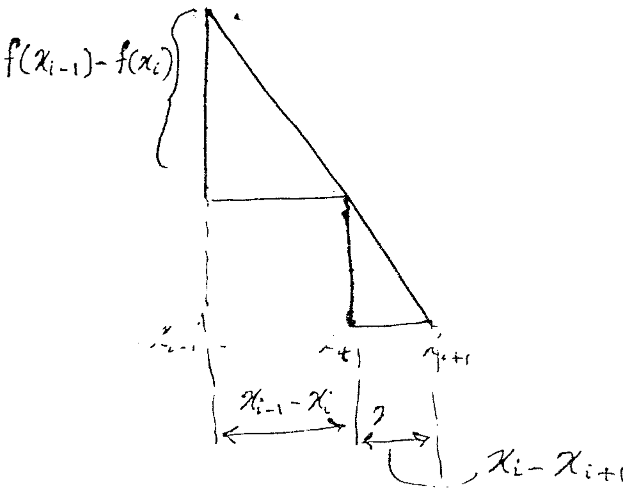
Midterm I May 2^{nd} 2009

Q2

Secant Method $\quad x_{i+1} = x_i - \dfrac{f(x_i)\cdot(x_{i-1} - x_i)}{f(x_{i-1}) - f(x_i)}$



مثلثم متشابهم



$$\therefore \quad \frac{f(x_{i-1}) - f(x_i)}{f(x_i) - 0} = \frac{x_{i-1} - x_i}{x_i - x_{i+1}}$$

$$x_i - x_{i+1} = \frac{f(x_i)\cdot(x_{i-1} - x_i)}{f(x_{i-1}) - f(x_i)}$$

$$x_{i+1} = -x_i + \frac{f(x_i)\cdot(x_{i-1} - x_i)}{f(x_{i-1}) - f(x_i)}$$

$$x_{i+1} = x_i - \frac{f(x_i)\cdot(x_{i-1} - x_i)}{f(x_{i-1}) - f(x_i)}$$

- $f(x) = \sqrt[m]{N}$

$$x = \sqrt[m]{N} \Rightarrow x^m = N \Rightarrow x^m - N = 0$$

$$\therefore f(x) = x^m - N \Rightarrow f'(x) = m \cdot x^{m-1}$$

$$\because \cancel{x_{n+1}} \; x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$\therefore x_{n+1} = x_n - \frac{x_n^m - N}{m \cdot x_n^{m-1}} \Rightarrow x_{n+1} = \frac{m \cdot x_n^{m-1} \cdot x_n - (x_n^m - N)}{m \cdot x_n^{m-1}}$$

$$= \frac{m \cdot x_n^{m-1+1} - x_n^m + N}{m \cdot x_n^{m-1}}$$

$$\therefore x_{n+1} = \frac{m \cdot x_n^m - x_n^m + N}{m \cdot x_n^{m-1}}$$

$$\therefore x_{n+1} = \frac{(m-1) \cdot x_n^m + N}{m \; x_n^{m-1}}$$

#

:) $f(x) = 2\sin(x) - e^{x/4}$ 4 - iterations

Carryout calculations up to 3 digits

$a = 0$; $b = 2$ $[0, 2]$

check that the root lies in $[0, 2]$

$f(a) = f(0) = -1$ -ve

$f(b) = f(2) = 0.07$ +ve

Thus the root lies in the interval $[0, 2]$

$c_1 = \frac{0+2}{2} \Rightarrow \boxed{c_1 = 1}$, $f(c_1) = 0.4$ +ve

$a_2 = a_1 = 0$, $f(a_2) = -1$

$b_2 = c_1 = 1$, $f(b_2) = 0.4$

$c_2 = \frac{0+1}{2} \Rightarrow c_2 = 0.5$, $f(c_2) = -0.17$ -ve

$f(a_2)$ and $f(c_2)$ are -ve

$a_3 = c_2 = 0.5$, $f(a_3) =$ -ve

$b_3 = b_1 = 1$

$c_3 = \frac{0.5+1}{2} \Rightarrow \boxed{c_3 = 0.75}$, $f(c_3) = 0.157$ +ve

$a_4 = a_3 = 0.5$

$b_4 = c_3 = 0.75 \Rightarrow c_4 = \frac{0.5 + 0.75}{2} \Rightarrow \boxed{c_4 = 0.625}$, $f(c_4) = 0.0011$

$b_5 = 0.625$

$c_5 = \frac{0.5+0.625}{2} \Rightarrow \boxed{c_5 = 0.5625}$

* Diagonally Dominant Matrix ≙ It is a matrix in witch any absolute value of element in the main diagonal is greater than the absolute value of the sum of the other elements

$$|a_{ii}| > \left|\sum a_{ij}\right| \quad i \neq j$$

* partial pivoting is interchanging either rows or columns to avoid division by zero or division by very small pivoting element

Singular Matrix ; It has a zero Determinant

:) $2x + y - 3z = -11$

$3x - 4y + 5z = 38$

$-2x + 3y + 7z = 15$

$a_{ii} \geq |a_{ij}| \quad i \neq j$

$$LU = A \Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix} \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix} = \begin{bmatrix} 2 & 1 & -3 \\ 3 & -4 & 5 \\ -2 & 3 & 1 \end{bmatrix}$$

$u_{11} = a_{11} = 2, \quad u_{12} = a_{12} = 1, \quad u_{13} = a_{13} = -3$

$l_{21} = \frac{a_{21}}{u_{11}} \Rightarrow \boxed{l_{21} = \frac{3}{2} = 1.5}, \quad l_{21} \cdot u_{12} + u_{22} = -4 \Rightarrow u_{22} = -4 - \frac{3}{2}$

$\boxed{u_{22} = -5.5}, \quad l_{21} \cdot u_{13} + u_{23} = 5 \Rightarrow u_{23} = 5 - \frac{3}{2}(-3)$

$\boxed{u_{23} = \frac{19}{2} = 9.5}$

$l_{31} = \frac{-2}{2} \Rightarrow \boxed{l_{31} = -1}, \quad l_{31} u_{12} + l_{32} \cdot u_{22} = 3 \Rightarrow l_{32} = \frac{3 - (-1)1}{-5.5}$ (-0.727)

$\boxed{l_{32} = -0.727}, \quad l_{31} \cdot u_{13} + l_{32} \cdot u_{23} + u_{33} = 7 \Rightarrow u_{33} = 7 - (-1)(-3) - \frac{4}{5.5}(9.5)$

$\cancel{u_{33} = 2.909}$ $\quad \boxed{u_{33} = 10.909}$

$$L = \begin{bmatrix} 1 & 0 & 0 \\ 1.5 & 1 & 0 \\ -1 & -0.727 & 1 \end{bmatrix}$$

$$U = \begin{bmatrix} 2 & 1 & -3 \\ 0 & -5.5 & 9.5 \\ 0 & 0 & 10.909 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 1.5 & 1 & 0 \\ -1 & -0.727 & 1 \end{bmatrix} \begin{bmatrix} Z_1 \\ Z_2 \\ Z_3 \end{bmatrix} = \begin{bmatrix} -11 \\ 38 \\ 15 \end{bmatrix}$$

$\boxed{Z_1 = -11}$, $1.5Z_1 + Z_2 = 38 \Rightarrow Z_2 = 38 - 1.5(-11)$

$\boxed{Z_2 = 54.5}$ $-Z_1 - 0.727Z_2 + Z_3 = 15 \Rightarrow Z_3 = 15 + Z_1 + 0.727Z_2$

$\boxed{Z_3 = 43.622}$ $Z_3 = 15 - 11 + 0.727(54.5)$

$UX = Z$

$$\begin{bmatrix} 2 & 1 & -3 \\ 0 & -5.5 & 9.5 \\ 0 & 0 & 10.909 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -11 \\ 54.5 \\ 43.622 \end{bmatrix}$$

$0.909x_3 = 43.622 \Rightarrow \boxed{x_3 \cong 4}$

$5x_2 + 9.5x_3 = 54.5 \Rightarrow x_2 = \frac{9.5 \times 4 - 54.5}{-5.5} \Rightarrow \boxed{x_2 = 3}$

$x_1 + x_2 - 3x_3 = -11 \Rightarrow x_1 = \frac{-11 + 3 + 3(4)}{2} \Rightarrow x_1 = 2$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ -3 \\ 4 \end{bmatrix}$$